

Concept of the Tension Truss Antenna

Koryo Miura* and Yasuyuki Miyazaki†

The Institute of Space and Astronautical Science, Yoshinodai, Sagamihara, Kanagawa, Japan

The concept of the tension truss antenna is proposed. It is shown that a parabolic reflector surface can be formed by a three-dimensional integrated cable lattice system that can be called a tension truss. The tension truss antenna consists of a triangular-faceted cable truss, the rf reflector surface, and a supporting structure which provides support as well as pretension of the cable lattice. The primary feature of the antenna is that its shape is virtually determined by geometric quantities such as the lengths and arrangement of cable members. Because of this feature, the adjustment of antenna surface can be done directly by changing lengths of cable members which form the surface. This adaptive nature of the concept is favorable for controlling of a reflector surface in orbit. It is also easily deployable and retractable, and varieties of facet sizing and arrangement are possible. Modeling of truss geometry, surface accuracy, analysis on formation of a parabolic surface, surface adjustment algorithm, and system design are investigated.

Nomenclature

| | |
|-----------------------------|---|
| D | = diameter |
| $d\mathbf{l}$ | = member deformation vector (i th component $dl_i = l_i - L_i$) |
| $d\mathbf{l}^*$ | = $[\mathbf{S}]^{-1} d\mathbf{x}$ |
| $d\mathbf{x}$ | = nodal displacement vector |
| $(dx_i, dy_i, dz_i)T$ | = nodal displacement vector of node (i) |
| $d\mathbf{x}^n$ | = nodal displacement vector for unit-length deformation of the n th member |
| $(dx_i^n, dy_j^n, dz_i^n)T$ | = i th nodal displacement vector as the n th member has deformed for unit length |
| $dz(8)$ | = target deviation of z coordinate of node (8) |
| EA | = axial stiffness of a cable |
| F | = focal length |
| H | = maximum deviation of the sphere from the plane of an equilateral triangle with vertices on the sphere |
| j | = total number of nodes excluding fixed points |
| j^* | = total number of joints |
| L | = length of one side of the triangle |
| L | = member length vector of the initial state (i th component L_i) |
| l | = member length vector in the deformed state (i th component l_i) |
| l_c | = $l - d\mathbf{l}$ |
| M | = number of sides of a pyramid |
| m | = total number of members |
| N | = number of subdivision of one side of a pyramid |
| P | = internal pressure |
| R | = radius of a sphere |
| R_1, R_2 | = principal radii of curvature |
| r | = number of restraints |
| $[\mathbf{S}]$ | = sensitivity matrix of the nodal displacement for the member deformation |
| s | = number of triangles mapped on the surface of a pyramid |

| | |
|----------------|--|
| T_1, T_2 | = principal tensile forces in the membrane |
| δ_{rms} | = root-mean-square deviation of the triangle from the sphere |

Introduction

VARIOUS structural concepts have been proposed for space-borne large deployable antennas. Among them, antennas constructed by rigid spatial trusses or beams are prevalent. The wrap-rib antenna and the octed-truss antenna are a few examples. As the size of antennas increases, however, lighter or larger ones constructed by rigid members will meet serious difficulties caused by increasing weight, packaged volume, and number of joints. So, the authors believe that antennas using primarily tensile members will play a major role in future lightweight, large space antennas. The hoop-column antenna is a cable lattice antenna in this category.

A basic form of cable lattice antennas is that each radial section of a parabolic surface is formed by a so-called catenary assemblage (Fig. 1a). The geometry of the catenary assemblage is determined by the equilibrium of forces among lattice cable members, or in other words the geometry is dependent on forces. For this reason, adjustment of the antenna surface, presumably by adjusting lengths of the cables, becomes a complicated procedure which usually does not converge quickly. Another basic form of cable lattice systems is based on the pretensioned truss concept as shown in Fig. 1b. Each radial section of a parabola is approximated by a pretensioned truss assemblage. Therefore, the shape of this cable lattice system is virtually determined by the lengths and arrangement of cable members and the effect of prestress is of secondary importance.

Both systems just mentioned, however, have inherent disadvantage owing to their radial arrangement of planar lattice assemblages. As the distance between adjacent radial assemblages increases in proportion to the distance from the center, the size of facet has to be large in the outer region of the reflector. Also, their low torsional stiffness is a problem. In short, such a system is nothing but an assemblage of planar cable lattices and it does not compose an integrated three-dimensional structure.

In this paper, the authors present a new concept of an antenna reflector composed of a three-dimensional integral cable lattice system. It may be called the concept of the tension truss antenna. The geometric modeling of tension truss antennas, the boundary restraints, the surface accuracy of polyhedral approximation, analysis of reflector surface deviation, the surface adjustment algorithm, and a feasibility model are described in this paper.

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*Professor and Director, Division of Spacecraft Engineering Research. Member AIAA.

†Graduate Student.

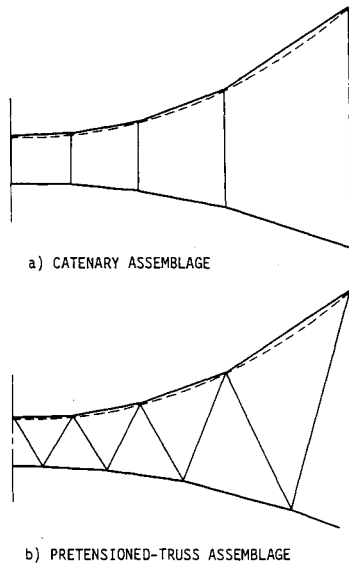


Fig. 1 Cable lattice structures for antennas.

Concept of the Tension Truss

The theme that constructing an antenna surface in three-space by a latticed tensile structure is, in terms of mathematics, establishing a finite set of lattice points describing a parabolic surface as close as possible. In this section, we will pursue a tensile structural system by which a finite set of lattice points in three-space is determined by a geometric complex.

Let us consider the simplest three-dimensional statically determinate truss, that is, a tetrahedral truss as shown in Fig. 2a. The four nodes of the truss provide a set of lattice points in three-space. We assume for the present that strains in truss members give negligible effect on spatial position of nodes. If each member of the truss is replaced by a flexible cable member of identical natural length, a cable lattice structure as shown in Fig. 2b is obtained. Then, if an appropriate force is applied to each node of the lattice, so that every cable member may be in the state of tension, a three-dimensional rigid structure results as shown in Fig. 2c. It is clear that lattice points determined by configurations in Figs. 2a and 2c are virtually identical with each other, because the lengths of corresponding members and their ways of combination are equal. In other words, a set of lattice points determined by a statically determinate truss can be realized by a cable lattice structure activated by proper forces exerted by a proper supporting structure. It should be noted that its shape is primarily determined by geometric quantities of lattice components. The applied forces exerted from a supporting structure are used for stretching cable members, and the effect of stretching on the geometry of the truss is always calculable and is of secondary importance. Thus, the concept of tension-activated cable lattice structure is established. Since this structure is in essence a truss rather than a lattice, it can be called a "tension truss."

The basic conditions for a tension truss are as follows:

- 1) Each member of the truss is made of a flexible cable and it must be in the state of tension in use.
- 2) The truss must be stable and statically determinate, that is,

$$m = 3j^* - r \tag{1}$$

and the stiffness matrix is not singular.

Concept of the Tension Truss Antenna

The application of the tension truss concept to deployable antennas is to be investigated in this section. Let us consider a geodesic truss dome, which is a kind of spatial truss, as shown

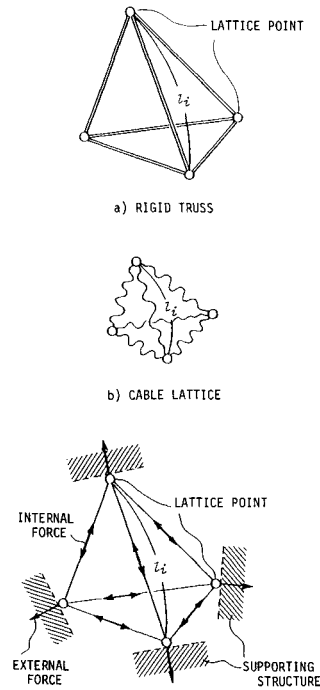


Fig. 2 Concept of the tension truss.

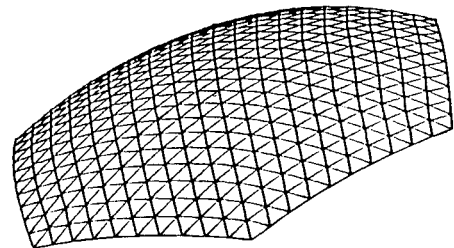


Fig. 3 A geodesic truss dome.

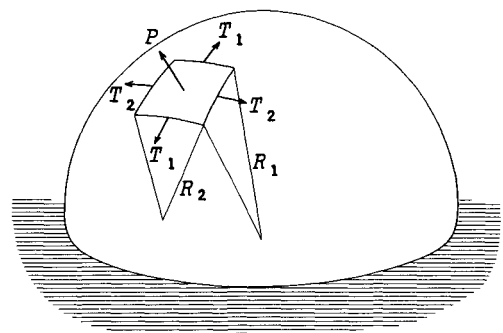


Fig. 4 Air-supported membrane structure.

in Fig. 3. This structure is an efficient lightweight truss and is erectable in space, so it becomes one of the candidates for future space antennas. If a geodesic truss dome can be realized by a deployable tension truss, it may be quite useful.

The principal problem of this approach is to find a method of setting every member of the truss in the state of tension. Some useful hints for solving this problem come from observation of an air-supported membrane structure such as an air dome. As shown in Fig. 4, an arbitrary unit element of the membrane is in the state of tension governed by the following

formulas:

$$T_1/R_1 + T_2/R_2 = P, \quad 1/R_1 \times 1/R_2 > 0 \quad (2)$$

The latter part of Eq. (2) means that the surface has to be the one with positive Gaussian curvature. Because the geodesic dome is also the surface of positive Gaussian curvature, the tensile forces may be produced everywhere in the truss if a force system simulating the internal pressure P of the air dome is provided. This is done by numbers of concentrated forces applied to nodes of the tension truss as shown in Fig. 5. These concentrated forces are supplied externally by a supporting structure. It should be noted that these applied forces are not necessarily of certain definite values and directions, since the necessary condition is that the internal forces of truss members are at least positive. The effect of stretching of members on the resulting surface can be calculated appropriately. The necessary boundary restraints can always be given, and the details are explained in the following section. Thus, the concept of tension truss antenna is established.¹

The most important feature of the tension truss antenna is that its shape is virtually predetermined by geometric quantities such as the lengths and arrangement of cable members of the truss. External forces are applied only for the purpose of giving the truss tensile forces to reproduce such predetermined shape. The effect of stretching of members on the shape is of secondary importance. Therefore, the concept of tension truss antenna is essentially different from those of current tension structure antennas such as the hoop-column antenna, whose shape is primarily determined by the equilibrium of forces. Another important feature is that its geometry is quite general and can meet varieties of requirements with regard to facet sizing and arrangement.

Geometric Modeling and Boundary Restraints

Since the concept of the tension truss antenna is quite general with regard to its truss geometry, there are various geometric schemes for subdividing a parabola to a lattice structure. It seems that the geometric scheme proposed by Nayfeh and Hefzy² is adequate and readily applicable to the present purpose; their scheme is adopted in this study. The mapping procedure is illustrated in Fig. 6. A parabolic surface is subdivided at the aperture circle into M equal elements. A line connecting these points to the center of the reflector forms the M -sided pyramid shown in Fig. 6a. Each side of the pyramid is then subdivided into N equal parts (bays) to form subelements (Fig. 6b). Finally, the points of intersection of these triangles are projected or mapped on the surface to obtain the final coordinates of the truss nodes (Fig. 6c).

The condition of statical determinacy and the necessary boundary restraints are now examined. The number of triangles s in this modeling is given by

$$s = M(2i + 1) = MN^2 \quad (3)$$

and the total number of members is

$$m = MN(1 + 3N)/2 \quad (4)$$

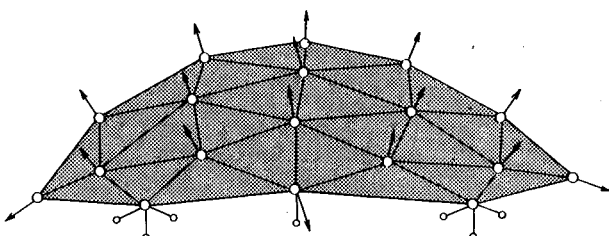


Fig. 5 Principle of the tension truss antenna.

and the total number of joints is

$$j^* = 1 + MN(1 + N)/2 \quad (5)$$

The necessary number of restraints r for the statically determinate truss is thus obtained from Eq. (1) as follows:

$$r = 3 + MN \quad (6)$$

The number of joints at the boundary is MN , and then there are $3MN$ degrees of freedom. Because $3MN$ is always larger than $r = 3 + MN$ ($M > 2$, $N > 0$), it follows that the necessary restraints can always be given by the joints located at the boundary.

Surface Errors Induced by Polyhedral Approximation

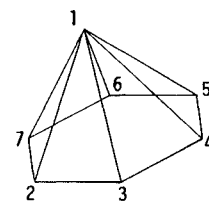
Among possible sources of errors for tension truss antennas, the basic one is due to the approximation of a parabolic surface by a polyhedral surface with triangular facets.

Agrawal et al.³ have discussed the errors induced by polyhedral approximation of such lattice shells and the result is equally applicable to the tension truss antenna. The following is some of their results. If the reflector under consideration is shallow, the facets tend toward equilateral triangles, and the principal curvatures are nearly equal for any surface of revolution. Thus, the δ_{rms} calculation for an equilateral triangle on a spherical surface should be a good approximation for the actual geometry. The maximum deviation H of a sphere of radius R from the plane of an equilateral triangle with vertices on the sphere is

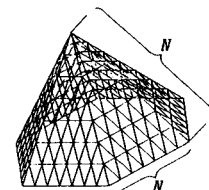
$$H = L^2/6R \quad (7)$$

The effective sphere is defined as that which demonstrates the δ_{rms} with respect to the triangular facet. The distance between the sphere containing the vertices and the effective sphere is $3/4H$. The corresponding δ_{rms} is

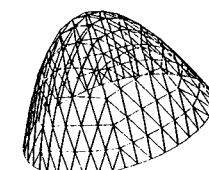
$$\delta_{rms} = (1/8\sqrt{15})L^2/R \quad (8)$$



a) M-SIDED PYRAMID



b) PYRAMID N SUBDIVISION



c) MAPPING FINAL PARABOLOIDAL SHAPE

Fig. 6 Geometric scheme for subdividing a parabola to a tension truss.

This value is calculated for a small conceptual model and a mission model, and the result is shown in Fig. 7.

Agrawal has also given a study on electromagnetic behavior resulting from triangular- and hexagonal-faceted reflection surfaces for a spherical reflector. He concluded that for larger antennas, triangular facet leads to the fewest number of structural members to meet accuracy requirements and acceptable electromagnetic performance. The results also suggest that the far-field patterns are governed by both the magnitude of the root-mean-square surface deviation and the periodic variation of the surface deviation over the reflector. As a result, slightly irregular facet geometries seem to result in better side lobe behavior than more regular patterns. In case of tension truss antennas, inclusion of slightly irregular facet geometries is easily provided without any structural penalties.

Analysis on Formation of Parabolic Surface

The inherent property of the tension truss antenna is that the structure is composed of members which are always tensioned and are unable to bear compressive forces. Considering this property, the manner by which the formation of a stable, accurate parabolic surface by an assembly of cable members has to be established. The result of the analysis in this direction is described in this section. It involves such items as the adjustment of node position, the effect of stretching of

members, effect of length deviation of a single member, and the effect of backup cable stiffness.

Model

The study was carried out about a mathematical model as shown in Fig. 8. It has diameter $D=1500$ mm, focal length $F=700$ mm, and it consists of 31 nodes, 9 fixed points, and 93 members. The stiffness of each cable truss member is $EA=10,000$ kg, and the applied forces at the boundary are those shown in Fig. 8. For comparison purpose, a computation was also carried out on an idealized tension truss antenna assuming infinite extensional stiffness of members. Hereafter, it is called a rigid tension truss in this paper.

Analytical Procedure

The heart of the present analysis is in essence to obtain the position of each node, under various conditions, when the natural (unstressed) lengths of truss members are given. As a matter of course, the analysis must take account of the effect of truss member stiffness. Accordingly, the equilibrium equations of tensions in truss cables and backup forces were formulated. In order to solve this nonlinear problem, the authors used the Newton-Raphson procedure.⁴

Adjustment of Node Position

The adjustment of each nodal position by changing lengths of truss members constitutes the basic procedure of formation of a parabolic surface by this concept. In case of a rigid tension truss, the adjustment of any nodal position can be done through changing lengths of members which are involved in the said node. Let us move node (8) in the z direction by as much as $dz(8)$, while the other nodal positions are unchanged (Fig. 9). The length changes are apparently required only to those members involved in node (8); i.e., the members (3), (5),

| model | F | D | F/D | R |
|------------------|-------|-------|------|-------|
| conceptual model | 700 | 1500 | 7/15 | 1400 |
| mission model | 10000 | 20000 | 0.5 | 20000 |

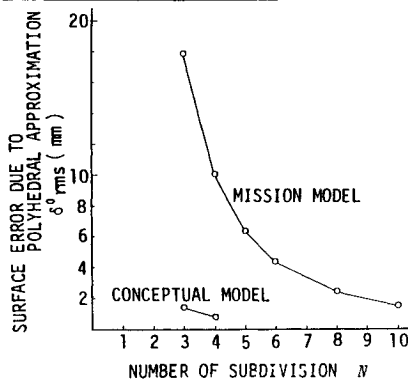


Fig. 7 Surface error due to the polyhedral approximation.

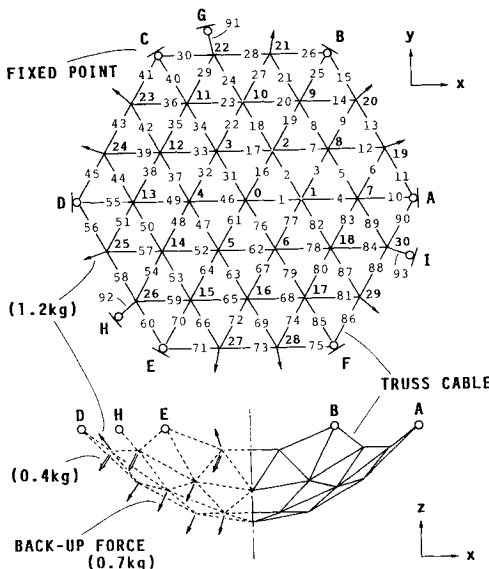


Fig. 8 Mathematical model A.

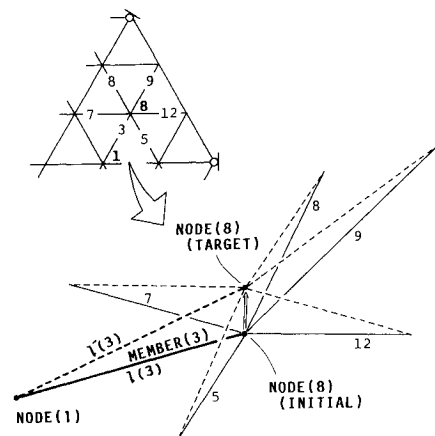


Fig. 9 Adjustment of nodal position by member length change.

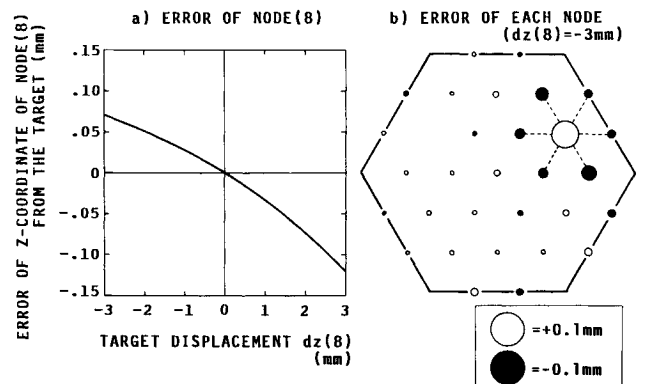


Fig. 10 Adjustment and resulting errors.

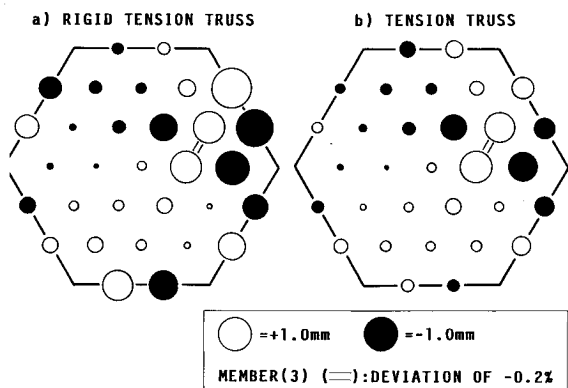


Fig. 11 Effect of a member length deviation.

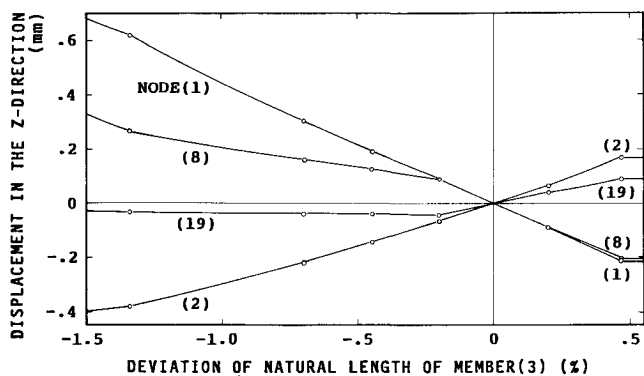


Fig. 12 Effect of increasing member length deviation.

(7), (8), (9), and (12). As shown in Fig. 9, the length of member (3) is to be extended by $l'(3) - l(3)$, where $l(3)$ and $l'(3)$ are the distances between node (8) and node (1) in the initial and the target state, respectively.

If the same adjustment procedure is applied to a real tension truss antenna, the stretching of truss members produces some amount of error in the position of nodes. The result of such a calculation is shown in Fig. 10. Figure 10a represents the error of the z coordinate of node (8) from the target when $dz(8)$ is given. Figure 10b shows the mapping of error of the z coordinate of each node induced by this adjustment for $dz(8) = -3$ mm. The white and black circled areas indicate proportionally the amounts of deviation of the z coordinate to the positive and negative directions, respectively. Note that these errors are zero for a rigid tension truss antenna. The largest error appeared at node (8); however, it is barely 2.35% of $dz(8)$. It is observed that the influence of the adjustment is minor at other nodes and is almost confined within the adjacent nodes. In view of practical application, this property suggests that the linear adjustment algorithm should be very effective.

Effect of a Member Length Deviation

The effect of deviation in natural length of a member on the position of various nodes is calculated.

Figure 11 shows the nodal displacements in the z direction for the case in which the natural length of member (3) is deviated as much as -0.2% . Figures 11a and 11b represent a rigid tension truss and a tension truss, respectively. It is observed that the effect is almost identical with both cases except at the boundary. Figure 12 shows the effect of increasing deviation of the length of member (3) on the displacements of several nodes in the z direction. As long as the length deviation is small, the linear relation holds between the given deviation and the nodal displacements. As long as the deviation increases further, however, the slackening of some members appears and the resulting nonlinear relation is observed in the figure.

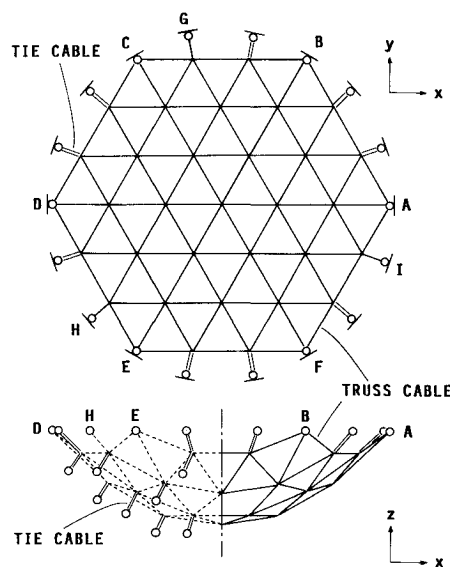


Fig. 13 Mathematical model B.

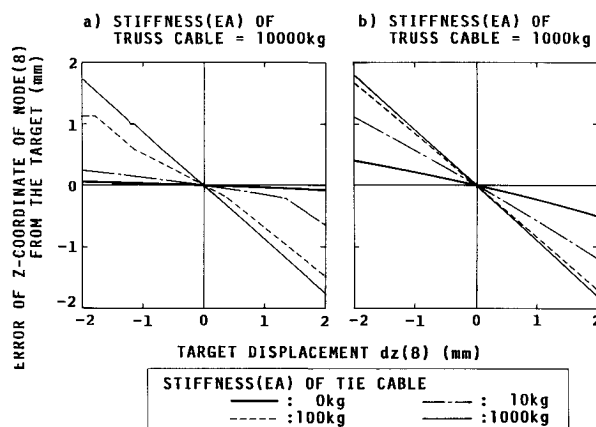


Fig. 14 Adjustment and resulting errors (model B).

Effect of Backup Cable

The model (model A) considered so far is the case where the nodes are tensioned through uniform forces (Fig. 8). Another model (model B), which is closer to practical designs, is now introduced (Fig. 13). In this case, the tension in the truss is supplied in a manner such that each node is connected with a fixed point by an elastic tie cable. The effect of stiffness of the truss and tie cables was estimated by this model. Figure 14 shows the error of the z coordinate of node (8) from the target when $dz(8)$ is given. The assumed stiffness of the truss cable is $EA = 10,000$ kg for Fig. 14a and $EA = 1000$ kg for Fig. 14b, while the stiffness of the tie cable is $EA = 0, 10, 100$, and 1000 kg. Here, the stiffness $EA = 0$ kg means that the tie cable has no stiffness, or in other words, the constant force is applied to the truss node, which corresponds to Fig. 10a. Figure 14 indicates that the combination of the rigid truss cable and extensible tie cables could produce favorable results from a viewpoint of accuracy. Note that the nonlinearity shown in Fig. 14a is caused by the slackening of some truss members.

Surface-Adjustment Algorithm—Adaptiveness

One of the most important feature of the concept is that the surface-shape adjustment can be done by changing the natural length of truss cable members. In order to perform the adjustment, an effective adjustment algorithm must be established because the antenna consists of a great number of truss members. However, if it is established, the antenna can be ad-

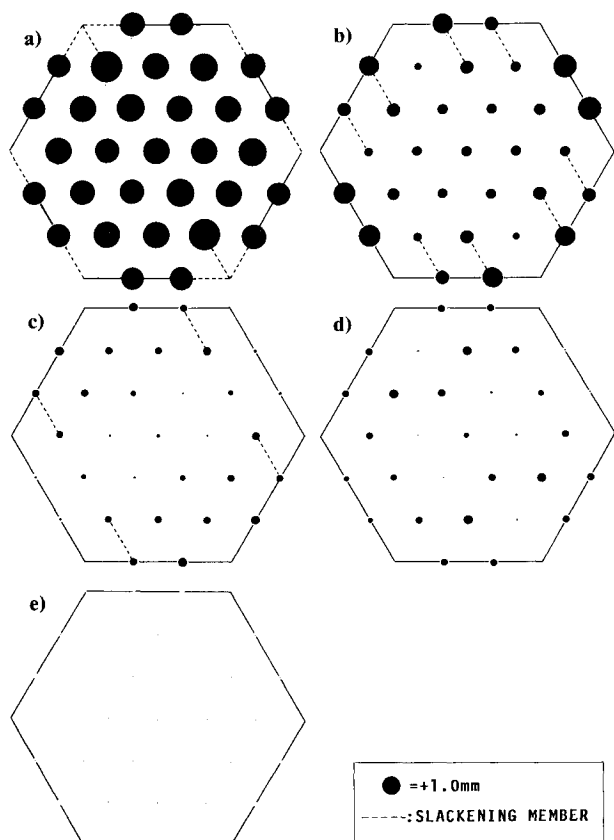


Fig. 15 Result of performing the surface-adjustment algorithm.

justed after fabrication and also after it is placed in orbit. Thus, the tension truss antenna can be considered as an adaptive structure.

In order to demonstrate this capability, the authors analyzed the following case: the natural lengths of truss members changed due to thermal effect and it caused deviation of nodal positions from the initial state. It is assumed that nothing is known in advance about the change in natural length of any member, but the deviation of each node is known. Under these conditions, the authors present an algorithm to adjust the nodal positions to the initial state by changing the member natural lengths. In this analysis, the method of linear sensitivity analysis was used, and the details are shown in the Appendix.

Figures 15a-15e show the adjusting process sequentially in four steps. In these figures, black circles indicate proportionally the nodal deviations and broken lines indicate slackening members. Figure 15a shows the initial state where deviation exists in almost every node, and there are some slackening members. After the first adjustment operation (Fig. 15b), the accuracy is greatly improved, although not complete. This is because the relationship between the member deformation and the nodal deviation is in fact nonlinear, while the adjustment is based on linear analysis. However, repeating this adjustment operation (Figs. 15c and 15d), the slackening members disappear and the nodal deviations become gradually smaller, and in the fourth adjustment (Fig. 15e), we have the correct answer.

Tension Truss Antenna System

The primary components of a tension truss antenna system are a reflector cable truss, rf reflecting surface, and a supporting structure. The rf reflecting surface can be provided by mesh materials stretched among triangular facets composed by cable truss members. The supporting structure provides the basis for supporting the reflector cable truss and stretching it

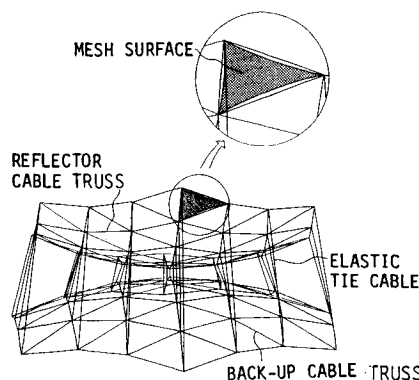


Fig. 16 Essential part of a tension truss antenna.

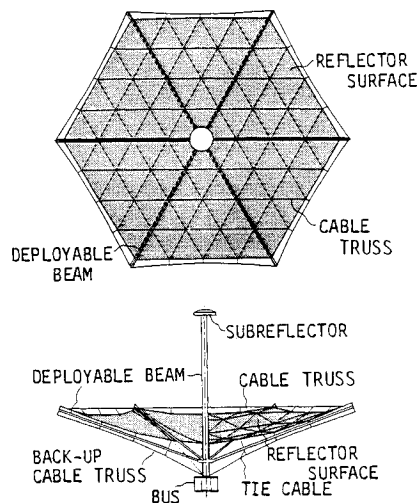


Fig. 17 System of a tension truss antenna.

through tie cables. The concept of the tension truss antenna is applicable to a variety of supporting structures, such as truss types, hoop-column types, radial rib types, and so on. The supporting structure itself can also be a cable truss structure (Fig. 16). In this figure, the external rigid support structure is not included. The illustration in Fig. 17 uses the radially arranged deployable beams and the cable truss as the supporting structure. In order to evaluate the concept of the tension truss antenna, a 3-m-diam model was fabricated. The surface accuracy measurement as well as the adjustment algorithm are being tested on the model, and the details will be reported in a separate paper.⁶

Concluding Remarks

The concept of the tension truss antenna is proposed and verified through analytical study. The most important feature of the antenna is that its shape is virtually determined by geometric quantities such as the lengths and arrangement of cable members. Because of these features, the adjustment of the antenna surface can be done directly by changing the lengths of cable members which form the surface. This adaptive nature of the antenna is favorable for controlling of a reflector surface in orbit. The tension truss antenna was selected as the 10-m diameter antenna for the space VLBI mission of The Institute of Space and Astronautical Science. The development of the engineering model and detailed analysis are in progress.

Appendix: Analysis of Surface-Adjustment Algorithm

Let the member length vector in the initial state and in the deformed state be expressed as

$$L = [L_1, L_2, \dots, L_m]^T \tag{A1}$$

$$l = [l_1, l_2, \dots, l_m]^T \quad (\text{A2})$$

where L_i and l_i are the i th member natural lengths are in the initial state and in the deformed state, respectively. Let the member deformation vector and the nodal displacement vector be expressed as

$$dl = [dl_1, dl_2, \dots, dl_m]^T \quad (\text{A3})$$

$$dx = [dx_1, dy_1, dz_1, dx_2, dy_2, \dots, dy_j, dz_j]^T \quad (\text{A4})$$

where $dl_i = l_i - L_i$ is the i th member deformation, and $(dx_i, dy_i, dz_i)^T$ is the i th nodal displacement vector. If the relationship between dl and dx is linear, it can be expressed as

$$dx = [S]dl \quad (\text{A5})$$

The sensitivity matrix of the nodal displacement for the member deformation is $[S]$. Here, $[S]$ is assumed as

$$[S] = [dx^1 | dx^2 | \dots | dx^m] \quad (\text{A6})$$

The vector dx^n is expressed as

$$dx^n = [dx_1^n, dy_1^n, dz_2^n, \dots, dy_j^n, dz_j^n]^T \quad (\text{A7})$$

where $(dx_i^n, dy_i^n, dz_i^n)^T$ is the i th nodal displacement vector from the initial state. Thus, the assumed matrix $[S]$ can be calculated when the initial state is defined.

Now, $[S]$ is a $3j \times m$ matrix. The tension truss is statically determinate ($3j = m$), so that $[S]$ is a nonsingular square matrix. Then, from Eq. (A5),

$$dl = [S]^{-1} dx \quad (\text{A8})$$

This equation shows that the member deformation vector dl , which is unknown in advance, is obtained from the given nodal displacement vector dx . Then, if the member natural

lengths are adjusted as

$$l \rightarrow dl_c = l - dl \quad (\text{A9})$$

l_c is equal to L , so that the nodal positions are identified with those in the initial state. In general, however, the relationship between dl and dx is nonlinear and Eq. (A5) is invalid. Thus, $dl^* = [s]^{-1} dx$ is not equal to dl and Eq. (A8) is also invalid. Then, if the member natural lengths are adjusted as

$$l \rightarrow l - dl^* = L + (dl - dl^*) \quad (\text{A10})$$

the nodal positions are not brought back into the initial state. If this adjustment procedure is repeated a few cycles, however, the nodal positions can be brought back into the initial positions. The result in Fig. 15 clearly shows that this method is quite effective.

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